Cartesian Solutions as Intersections

In the Cartesian plane, one of the most efficient (and visually meaningful) ways to find a solution to a system of equations is to graph the equations and find the points of intersections.

Where the graphs cross, the solution lies.

Done. Easy. Awesome.

Polar Solutions as Intersections

In the polar coordinate plane, intersections are not always what they seem. Sometimes it *appears* that two graphs cross, but they don't!

Wait… What?

Today we'll learn to distinguish between "real" and "fake" intersections in polar systems.

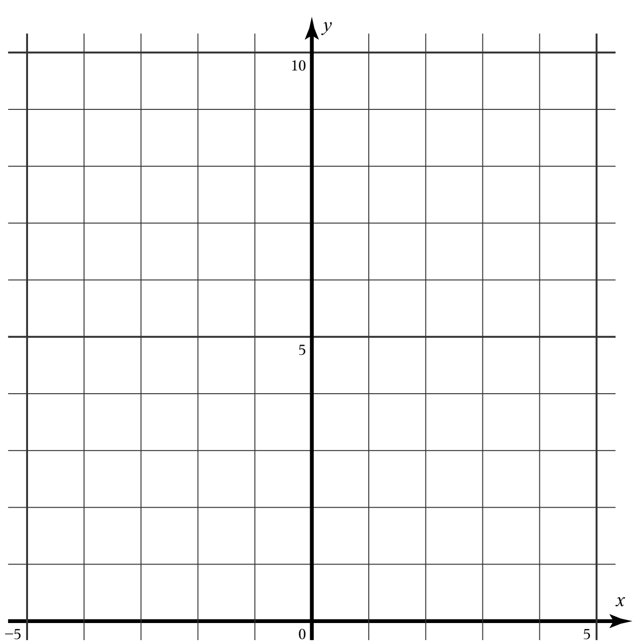
Solve This (Cartesian) System

Put your calculator in function mode. Graph these equations.

$$f1(x) = 6 - x$$

$$f2(x) = x^2$$

1. Sketch the graphs:



2. The solution in quadrant I is: $\left(\_\_\_\_,\_\_\_\_\right)$

3. The solution in quadrant II is: $\left(\_\_\_\_,\_\_\_\_\right)$

How do we know if a solution really is a solution? Substitute the (x, y) values into the original equations. If you get all true statements, then the solution is legit.

4. Confirm your two solutions. Show the substitution in the space below.

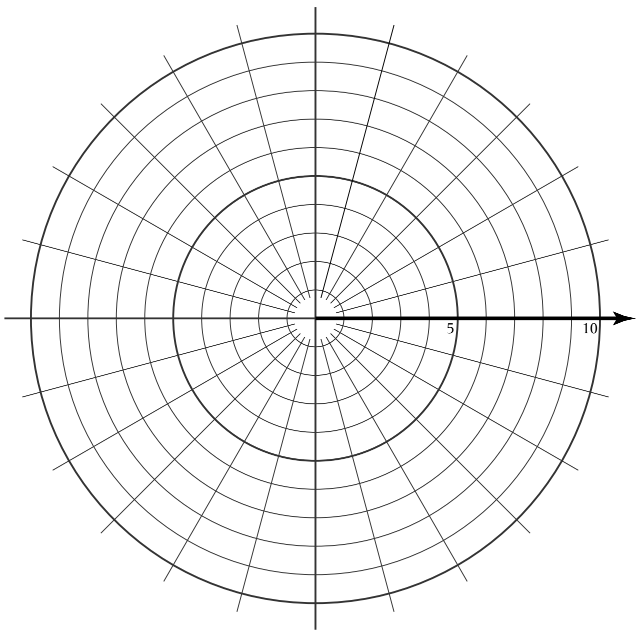
Solve This (Polar) System

Add a new graphs page. Put your calculator in polar mode. Graph these equations. (Make sure your *graphing angle* is set to degrees.)

$$r_1 = 3 + 2\cos\theta$$
$$r_2 = 5\sin\left(2\theta\right)$$

Use a domain of $0 \le \theta \le 360$ and a θ −step of 2 (that's right, use 2, not 5!).

5. Sketch the graphs:



6. It looks like there are eight points of intersection. Label these points P_1 through P_8 (starting with the "intersection" around 35º).

7. Estimate the θ values for each of the points. Fill in your estimates in the table (Column 1).

8. Now use the trace feature on your calculator to find the **more precise values** of θ and r (for r_1 only!) for each potential point of intersection. Before you start, set the *trace step* to 1. Round each r value to the nearest tenth. Put the values in the Columns 2, 3.

To trace a different graph, press the up or down key.

9. Now trace to find the r values (of r_2) for each θ value from Column 2. Again, round each r value to the nearest tenth. Put the values in Column 4.

	θ **(Est.)**	θ **(Trace to 1º)**	r_1 **(Trace to 0.1)**	r_2 **(Trace to 0.1)**
P_1				
P_2				
P_3				
P_4				
P_5				
P_6				
P_7				
P_8				

What is true of a solution to a system in the Cartesian plane? Multiple functions whose graphs contain the same exact ordered pair:

$$\left(x, y\right)$$

So what is true of a solution (a *true intersection)* in the polar coordinate plane? Multiple functions whose graphs contain the same exact ordered pair:

$$\left(r, \theta\right)$$

10. With that in mind, which of the eight points are actual solutions? Explain how you know.

Auxiliary Cartesian Graphs to the Rescue

If I were a student in my class, I'd be thinking…

Isn't there an easier/better/more efficient way to find actual solutions of polar systems?

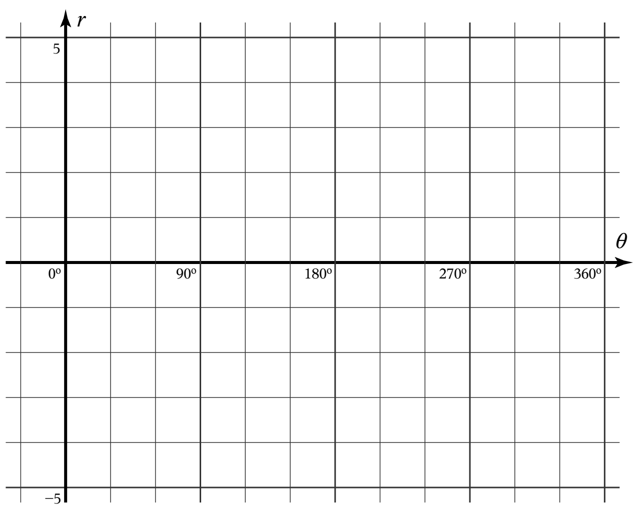
Say hello to… **auxiliary Cartesian graphs**!

Put your calculator in function mode. Graph these equations (in degree mode).

$$y = 3 + 2\cos x$$
$$y = 5\sin\left(2x\right)$$

Use a window with $x \in \left[0, 360\right]$ and $y \in \left[-6, 6\right]$.

11. Sketch the graphs:



Wait a second… These don't look like the graphs we've been working with. What's going on?!?!

12. Use your calculator to find the intersection points. Write them below. Label them on the graph (A, B, C, D).

13. Now compare the original polar graphs (the limaçon and the rose) with the auxiliary Cartesian graphs. They're **different** graphs to be sure, but there is an important connection. What's the connection? (Bonus: What's going on with the "fake" points of intersection?)

14. What did you learn today that you didn't already know?

15. What are you still confused about?