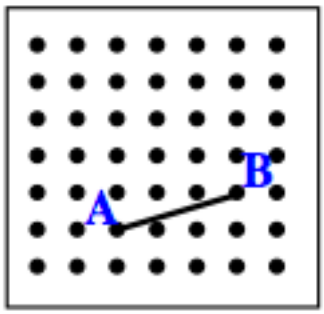
Task 1

Make an isosceles triangle $\triangle ABC$ on your geoboard using $\overline{AB}$ as one of the sides. How many different isosceles triangles can you make on your geoboard with $\overline{AB}$ as one of the sides?[1]



[1] Source for all tasks: http://mathpractices.edc.org/

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

- ☐ 1 Make sense of problems and persevere in solving them.
- ☐ 2 Reason abstractly and quantitatively.
- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
- ☐ 6 Attend to precision.
- ☐ 7 Look for and make use of structure.
- ☐ 8 Look for and express regularity in repeated reasoning.

Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. How would the results of the original problem change if you were allowed to put more than one geoboard together?

2. If you are free to move point *A* (leaving point *B* where it is), where would you move *A* to maximize the number of possible locations for point *C* on the geoboard? What if you could move point *B* (leave point *A)*?

3. Let's say that you can move both points, but had to leave them exactly the same distance from each other as they are now. In what position(s) can you place them to maximize the number of isosceles triangles that can be made?

4. In what position(s) can you place $\overline{AB}$ to *minimize* the number of isosceles triangles that can be made?

5. Imagine the exact same problem posed on another geoboard of the exact same area, but not the exact same shape. How would that affect the number of possible isosceles triangles?

6. How does the problem change if we want an equilateral triangle instead of isosceles? What if we want to guarantee a scalene triangle?

Task 4

Two siblings—a brother and a sister—attend the same school. Walking at constant rates, the brother takes 40 minutes to walk home from school, while the sister takes only 30 minutes on the same route. If she leaves school 6 minutes after her brother, how many minutes has he traveled before she catches up to him?

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

- ☐ 1 Make sense of problems and persevere in solving them.
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- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
- ☐ 6 Attend to precision.
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- ☐ 8 Look for and express regularity in repeated reasoning.

Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. Two siblings attend the same school. The brother takes 40 minutes to walk home, while his sister takes 30 minutes. If they arrive home at the same time, what portion of the distance had he covered when she left school?

2. Two siblings attend the same school. The brother takes 40 minutes to walk home, while his sister takes 30 minutes. If they met at the halfway point, how long after he left school did she start walking home?

3. Two siblings attend the same school. On the trip to school from home, the sister walks twice as fast as the brother does and leaves 6 minutes later.

   A. Is it possible to determine at what time they meet? If so, find it. If not, explain why not.

   B. Is it possible to determine the distance at which the brother and sister meet? If so, find it. If not explain why not.

4. Now, the two siblings attend different schools. The sister's school is 1 mile farther from home than brother's, and she takes 50 minutes to get home while he only takes 30. If they are both walking at the same speed, find the distance to each school and the speed they are walking.

Task 5

Make up a set of eight numbers that simultaneously satisfy these constraints:

Mean: 10
Median: 9
Mode: 7
Range: 15

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

- ☐ 1 Make sense of problems and persevere in solving them.
- ☐ 2 Reason abstractly and quantitatively.
- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
- ☐ 6 Attend to precision.
- ☐ 7 Look for and make use of structure.
- ☐ 8 Look for and express regularity in repeated reasoning.

Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. Make up an additional set of eight numbers that also simultaneously satisfy these constraints:

   Mean: 10
   Median: 9
   Mode: 7
   Range: 15

2. A particular school has measured the height, to the nearest half–inch, of all the boys in one seventh–grade class and one eighth–grade class. (A) Which measure—median or range—would you expect to change more from one grade to the next? (B) Which measure—mode or mean—is likely to be more informative about the height of a student selected at random from one of the grades?

3. Make up a set of eight numbers that simultaneously satisfy these constraints.

   Mean: 10
   Median: 10
   Mode: 7
   Range: 15

4. Make up a set of eight numbers that simultaneously satisfy these constraints.

   Mean: 9
   Median: 10
   Mode: 7
   Range: 15

Task 6

If Raj has 3 times as many one–dollar bills as he has five–dollar bills, what are possible amounts of money he could have? Could he have $40? Could he have $42? What do you notice about the amounts he could have?

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

- ☐ 1 Make sense of problems and persevere in solving them.
- ☐ 2 Reason abstractly and quantitatively.
- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
- ☐ 6 Attend to precision.
- ☐ 7 Look for and make use of structure.
- ☐ 8 Look for and express regularity in repeated reasoning.

Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. Carmen has 2 five–dollar bills and 3 one–dollar bills, and Kareem has 3 five–dollar bills and 2 one–dollar bills. Who has more money?

2. If Raj has a total of $120, how many of each bill does he have?

3. What if Raj has **four** times as many one–dollar bills as he has five–dollar bills? What possible amounts of money could Raj have under this condition?

4. What if Raj has **seven** times as many one–dollar bills as he has five–dollar bills? What possible amounts of money could Raj have under this condition?

5. What if Raj had *a* times as many one–dollar bills as he has five–dollar bills? What possible amounts of money could Raj have under this condition?

6. What if Raj had **four** times as many one–dollar bills as he has **fifty**–dollar bills? What possible amounts of money could Raj have under this condition?

7. What if Raj has **four** times as many **ten**–dollar bills as he has **fifty**–dollar bills? What possible amounts of money could Raj have under this condition?

8. What if Raj has **seven** times as many **hundred**–dollar bills as he has **fifty**–dollar bills? What possible amounts of money could Raj have under this condition?

9. What if Raj has *a* times as many **hundred**–dollar bills as he has **fifty**–dollar bills? What possible amounts of money could Raj have under this condition?

Task 9 • Part 1

Suppose the post office only sold five–cent stamps and seven–cent stamps. Some amounts of postage can be made with just those two kinds of stamps. For example, 1 five–cent and 2 seven–cent stamps make 19 cents in postage, and 2 five–cent stamps makes 10 cents in postage. Which amounts of postage is it impossible to make using only five–cent and seven–cent stamps?

Task 9 • Part 2

Suppose the post office only sold six–cent and nine–cent stamps. Which amounts of postage is it impossible to make?

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

- ☐ 1 Make sense of problems and persevere in solving them.
- ☐ 2 Reason abstractly and quantitatively.
- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
- ☐ 6 Attend to precision.
- ☐ 7 Look for and make use of structure.
- ☐ 8 Look for and express regularity in repeated reasoning.

Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. If the post office has only two–cent stamps and three–cent stamps, what amounts of postage cannot be made?

2. If the post office has only three–cent stamps and five–cent stamps, what amounts of postage cannot be made?

3. If the post office has only four–cent stamps and nine–cent stamps, what amounts of postage cannot be made?

4. Suppose the post office only sold six–cent and nine–cent stamps. What amount of postage can be made?

5. Suppose the post office only sold m–cent stamps and n–cent stamps. Suppose also that, above some amount of postage, all amounts of postage can be made. What can you say about m and n?

6. What was the largest impossible amount of postage in each of the questions 1–3? For m–cent stamps and n–cent stamps in which all postage after a certain point can be made, what is the largest impossible amount of postage in terms of m and n?

7. How would the original task from the dialogue change if you could buy a negative number of stamps? (This can also be thought of as buying stamps worth –5 or –7 cents.) Which amounts of postage are impossible to make using only five–cent and seven–cent stamps? Which amounts of postage are impossible to make using only six–cent and nine–cent stamps?

Task 10

Find the dimensions of all rectangles whose area and perimeter have the same numerical value.

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

- ☐ 1 Make sense of problems and persevere in solving them.
- ☐ 2 Reason abstractly and quantitatively.
- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
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- ☐ 8 Look for and express regularity in repeated reasoning.

Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. Find all the pairs of positive unit fractions whose sum is $\frac{1}{2}$. How does this problem compare with the rectangle problem the students are working on in the dialogue?

2. Are there any triangles whose perimeter and area share the same numeric value? Circles? Pentagons? Hexagons? Is there an analogous equation for any *n*–gon?

3. Are there any rectangular prisms whose surface area and volume share the same numeric value? Spheres? Triangular prisms? Other 3–D shapes?

4. Find all the groups of three positive unit fractions (not necessarily all distinct) that add to $\frac{1}{2}$.

Task 13

Asher is selling carnation flowers. He makes bouquets of different sizes by using 4 white carnations for every 3 pink carnations. How many pink carnations are there in a bouquet with t total carnations?

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

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- ☐ 2 Reason abstractly and quantitatively.
- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
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Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. Using the same task as in the dialogue, how many white carnations are in a bouquet of 14 total flowers? 28 total flowers? 70 total flowers? *t* total flowers?

2. Using the same task as in the dialogue, does $t - \frac{t}{7} \cdot 3$ represent the number of white carnations in a bouquet of *t* total flowers? Explain why or why not.

3. If Asher were making bouquets of carnations using a ratio of 6 white carnations to 3 pink carnations, would the total number of flowers in each bouquet have to be a multiple of 9? Why or why not?

4. It takes Hebert 2 hours to water 5 gardens.

   A. How many gardens can he water in 4 hours? In 8 hours? In 40 hours? In *h* hours?

   B. For each question in part A, you used a repeating process. Explain what is happening in each step.

5. Answer the following:

   A. How many m/s in 10 km/hr? in 25 km/hr? In *x* km/hr?

   B. For each question in part A, you used a repeating process. Explain what is happening in each step.

6. In the 2012 Olympics, the United States won 46 gold medals and Russia won 24 gold medals. At the time, the United States had a population of approximately 314 million while Russia had a population of approximately 143 million. Based only on the information given, which country do you think was most successful at the Olympics and why?

Task 15

What is $64^{1/2}$?

What is $64^{1/3}$?

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

- ☐ 1 Make sense of problems and persevere in solving them.
- ☐ 2 Reason abstractly and quantitatively.
- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
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Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. What is $64^{1/6}$? Explain your answer.

2. What is $8^{1/2}$? Explain your answer.

3. What is $7^{1/4}$? Explain your answer.

4. In line 7, Chris makes the argument that $64^{1/2}$ cannot be the same thing as $64 \cdot \frac{1}{2}$. By analogy, we can make the claim that $64^{1/3}$ is not the same as $64 \cdot \frac{1}{3}$. Is there ever a time when a number raised to the one-third power *is* the same as multiplying the original number by one third?

5. In this dialogue, students are trying to make sense of rational exponents. For this task, consider how might you calculate a negative exponent? Try to come up with a value for 10^{-3} and support your answer.

Task 17

Two vertices of a triangle are located at *(0, 4)* and *(0, 10)*. The area of the triangle is 12 square units. Where is the third vertex located?

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

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- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
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Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. Two vertices of a triangle are located at $\left(-1,-5\right)$ and $\left(7,-5\right)$. The area of the triangle is 24 square units. Where can the third vertex be?

2. A point is located 5 units away from the point $\left(7,8\right)$. Where can the point be located?

3. Two vertices of a triangle are located at $\left(4,1\right)$ and $\left(4,5\right)$. The perimeter of the triangle is 12 units. Where can the third vertex be located? What tools might help you explore this situation?

Task 18

Given three vertices of a parallelogram at *A(1, 2)*, *B(4, 1)*, and *C(5, 3)*, where can the fourth vertex be located.

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

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- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
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Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. Find the coordinates of all other possible fourth vertices of the parallelogram described in the problem.
2. How do you know you have found all the possible fourth vertices? Justify your answer.
3. Given points $\left(2,1\right)$, $\left(6,1\right)$, and $\left(2,-2\right)$, how many parallelograms can you make? How many rectangles? Justify your answer.
4. Name three points that cannot be the vertices of a parallelogram, no matter where the fourth point is. What must be true about these three points?
5. Four keys are required to open a treasure vault. Your ripped map shows the location of three of the keys. You know the four keys are located at the corners of a parallelogram. Where could the fourth key be?

Task 19

How can we find the value of 2^{-3} and other expressions with negative exponents?

Standards for Mathematical Practice

Which of the eight CCSS Standards for Mathematical Practice did you use while solving this problem?

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- ☐ 3 Construct viable arguments and critique the reasoning of others.
- ☐ 4 Model with mathematics.
- ☐ 5 Use appropriate tools strategically.
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Student Dialogue and Teacher Reflection

What evidence do you see of students engaging in the Standards for Mathematical Practice?

Related Mathematics Tasks

1. What rule can be written to define raising a number to the zero power? Is this rule always true? Provide two different explanations.

2. Prove that $\left(x^m\right)^n = x^{m \cdot n}$.

3. Students in the dialogue (line 2) noticed that -2^3 and $\left(-2\right)^3$ both equal -8. Does $-2^4 = \left(-2\right)^4$? How about -5^3 and $\left(-5\right)^3$ or -5^4 and $\left(-5\right)^4$? In what circumstances does $-a^n = \left(-a\right)^n$?